NEGATIVE RESISTANCE PARAMETRIC OSCILLATOR

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[illegible]

UNITED STATES
NAVAL POSTGRADUATE SCHOOL

THESIS

A NEGATIVE RESISTANCE
PARAMETRIC OSCILLATOR

by

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Lieutenant, United States Navy

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Submitted in partial fulfillment of
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MASTER OF SCIENCE
IN
ENGINEERING ELECTRONICS

United States Naval Postgraduate School
Monterey, California

1960

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ABSTRACT

This paper is concerned with an investigation of [illegible] resistance parametric oscillator. As with [illegible] voltage variable capacitor, most commonly used in parametric amplifiers can be made to exhibit a negative resistance under certain circuit configurations. This negative resistance effect can be made to result in an oscillatory condition in the output of the device.

To accomplish the desired negative resistance the parametric device must be operated under the condition where the desired output frequency is the numerical difference between driving frequency and the [illegible] idle frequency, i.e.

$$f_s = f_p - f_i$$

Within the device this frequency relationship must always exist. [illegible] then conjectures as to what differential relation exists between the frequencies when external factors cause the driving power frequency to vary slightly. From the above frequency relation we know that the resultant frequencies must maintain the same additive relationship or that,

$$\Delta f_s = \Delta f_p - \Delta f_i$$

The specific concern of this paper is what parameters, if [illegible] determine the absolute shifts in frequencies and to satisfy the fundamental relation among the frequencies. One might conjecture that the individual shifts in frequency would be in proportion to the original frequencies such that,

$$\frac{\Delta f_n}{f_n} = \frac{\Delta f_p}{f_p}$$

or the ratio of the detuning ratios

$$\delta_{np} = \frac{\Delta f_n}{f_n} \Big/ \frac{\Delta f_p}{f_p} = 1$$

however, there is no theory as yet which supports this re[illegible]

From an idealized standpoint it could be possible to arrange the individual circuits such that one circuit, f_i, could [illegible] change of the driving frequency. While the remaining fre[illegible] remain relatively constant.

By the use of small signal circuit analysis and a [illegible] signal analysis it is shown that the ratio in the shifts [illegible] are related to the ratio of the frequencies with respect to [illegible] force and are also a function of the circuit Q's at th[illegible] condition. Also it is noted that in the case of a large [illegible] that a circuit detuning effect will result from a combina[illegible] three voltages, V_p, V_s, and V_i, present across the varia[illegible] This detuning process is a function of the driving power [illegible] the circuit and the resulting oscillation power.

Experimental results using an X-band driving force of [illegible] output oscillation of 900 mc indicate that the relative [illegible] the frequency of oscillation is unpredictable even under [illegible] circuit parameters largely as a result of the circuit detuning [illegible] exerted by the individual voltages across the diode when [illegible] goes into oscillation.

TABLE OF CONTENTS

Glossary of Terms

C_o - Diode capacitance at zero bias voltage

C_{bo} - Diode capacitance at bias voltage V_o

G_{dn} - Equivalent diode conductance at ω_n

ω_1 - Resonant frequency of signal circuit

ω_2 - Resonant frequency of idle circuit

ω_3 - Resonant frequency of driving circuit

F_1 - Diode filling factor of signal cavity

F_2 - Diode filling of idle cavity

F_3 - Diode filling factor of driving cavity

Q_1 - Q of unloaded signal cavity

Q_2 - Q of unloaded idle cavity

Q_3 - Q of unloaded driving cavity

Q_{dn} - Q of diode at ω_n

δ_n - Fractional deviation of frequency from resonance for circuit n.

$\delta_{\delta p}$ - Ratio of δ_1 / δ_3

Θ_2 - Phase angles of voltage at ω_2

Θ_3 - Phase angles of voltage at ω_3

Φ - Diode junction diffusion potential

$*$ - Complex conjugate

Introduction

In the relatively short period of two or three years great strides have been made in the development of parametric devices. Though the theory of operation and the numerous applications have been recognized for many years[1-2] the development of devices implementing the parametric effect had been largely experimental models used to substantiate the theory. The technology required to produce a device that had the characteristics of a time varying reactance had not advanced sufficiently far so that the practical parametric device could be utilized in electronic circuitry.

The ultimate development of the practical variable reactance device which could be used most effectively to produce desired parametric effect was the result of necessity and advances in technology.

Detection and communication systems are constantly being required to extend their effective ranges to meet the needs of both the military and non-military users. Output power and physical size of these systems have almost reached their limit both economically and practically. There existed, therefore, an immediate need to improve the range of both the present systems and those of the future by means other than increasing the size and power. The low noice characteristics of the parametric amplifier and its relative simplicity afforded a means of improving the systems and yet required no extensive increase in physical size. Hence, there existed an immediate need for the development of a practical parametric device.

The advance which ultimately led to the development of practical parametric devices was made in the development of semiconductor devices. With the rapid development of the transistor as a circuit

element came a thorough knowledge of both the theory and fabrication of semiconductor devices. Coupled with the need for a practical device with which to develop the parametric amplifier this knowledge led to the realization of a component which could produce the time varying non-linear reactance effects desired. The result is the variable capacitance diode which has now replaced the variable inductance ferrites, used previously as the principal means of obtaining the time varying reactance desired.

Briefly the variable capacitor is a semiconductor diode in which the width of the charge depletion area separating the P and N areas is caused to vary by application of voltage, much like physically separating the plates of a capacitor. Since little conduction current flows across the charge depleted area the effect is one of a displacement current rather than a conduction current, which results in the time varying capacitance characteristics when an AC voltage is applied.

Currently there are numerous parametric amplifiers in service both as modifications to present day equipment and as integral parts of proposed systems. As the commercial amplifiers become more and more common it is to be expected that the devices will find additional applications in the field of television and high frequency radio communication systems.

The basic operation of parametric devices is characterized by two unique operations. One is that the energy supplied to the system is obtained from a high frequency source rather than from a DC source. Secondly, the entire system as noted previously, is based on a time-varying energy storage element which when referred to subsequently will be understood to be a variable capacitance diode. Numerous electrical

and mechanical analogies of the action that takes place in the parametric device which makes possible the amplification effect have been described in the literature and will not be repeated here.[3,4]

While the presence of the nonlinear capacitance in a circuit produces components at all the frequencies $mf_p \pm f_s$, $(m = 0, 1, 2, 3, \cdots)$, the common representation of the perfect parametric device is that of a three frequency device where:

$$\omega_p = \omega_p \pm \omega_i \tag{1}$$

and ω_s = angular frequency of the signal

ω_p = angular frequency of the pumping power

ω_i = sum or difference angular frequency (commonly referred to as the idle frequency)

The particular configuration of frequencies that will be studied in the paper will be that where,

$$\omega_s = \omega_p - \omega_i \tag{2}$$

because, it is in this particular configuration that the device exhibits the negative resistance characteristic necessary for oscillation to occur. In addition it can be noted when used as an amplifier it is in this configuration that the device realizes the maximum gain and also the highest degree of instability.

The basic criteria for an oscillator, be it of parametric origin or of the conventional type, is that of stability. In the present types of oscillators the stability of oscillation is a function of temperature, power supply stability, and reliability of components. In the parametric oscillator the output frequency is a function of the driving frequency, namely that

$$\omega_s = \omega_p - \omega_i$$

That this frequency relationship must be maintained always is a fundamental characteristic of the device; hence, it is equally true that

$$\Delta\omega_p = \Delta\omega_s + \Delta\omega_i$$

$$\text{or} \quad \Delta\omega_s = \Delta\omega_p - \Delta\omega_i$$

The absolute value of $\Delta\omega_s$ is not defined. It is the purpose of this paper to explore the controlling factors, if any, which determine the actual shift in the output frequency resulting from fluctuation, in the driving force. Conceivably it could be that $\Delta\omega_s$ could be made to approach zero and $\Delta\omega_i$ approach the total changes in pu,ping frequency caused by external affects.

If the frequency shift of the output signal can be made infinitely small with respect to the deviations in the pumping frequency then it would be possible to construct oscillators of a high degree of stability from what could be described as secondary sources. It could result that parametric devices would have an important application as local oscillators in system that require precise measurements of time or frequency.

1. Theory of Operation

In the development of the parametric devices using nonlinear variable capacitor diodes, certain fundamental relations exist between the frequencies employed and the production of the negative resistance desired for oscillation. These relations are stated in practically all literature concerning parametric devices. In this paper we will use the definition given by Manley and Rowe[5,6] of the inverting negative conductance amplifier. Regardless of the particular definition used the frequency spectrum of the device to be used as an oscillator must be such that the frequency of the driving voltage, power supply, must be equal to the sum of the other two frequencies allowed to be present in the device such that

$$f_p = f_s + f_i$$

where f_p = the pumping or driving voltage

f_s = the desired signal frequency

f_i = the "idle" frequency*

The principal development will be along the lines of small signal analysis for stability relations while a corresponding large signal analysis will be used to investigate the resulting efficiencies and power levels both to commence oscillation and for useful output.

* "Idle" is the frequency at which power is neither supplied to nor removed from the device.

A. Theoretical Considerations

1. Small signal analysis:

The oscillatory consideration will be considered under the small signal analysis outlined by H. E. Rowe[6] and A. Van Der Ziel[7] for the nonlinear capacitor where the charge, q, is a functional of the applied voltage, v, such that

$$q = f(v)$$

and

$$dq = f'(v)\, dv \tag{3}$$

In the small signal analysis it is considered that the pumping voltage, V_p, is very much larger than the signal voltage, V_s and the idle voltage, V_i. Consequently, it will be assumed that

$$dq = f'(v_p)\, dv_p$$

since

$$Q = C V$$

The term $f'(v_p)$ may be considered as a periodic time varying capacitance of frequency f_p or

$$dq = C(v_p)\, dv_p \tag{4}$$

The periodic varying capacitor can then be expressed in a Fourier series,

$$C(v_p) = \sum_{n=-\infty}^{\infty} C_n\, e^{-j2\pi f_p t}$$

$$C_n = \frac{1}{2\pi} \int_0^{2\pi} f(v_p)\, e^{-j2\pi n f_p t}\, dt$$

Under the ideal conditions of only allowing the power to flow at frequencies, f_1, f_2 and f_3 we have for the inverting case that

$$\begin{vmatrix} Q_s \\ Q_i^* \end{vmatrix} = \begin{vmatrix} C_0 & C_1 \\ C_1 & C_0 \end{vmatrix} \begin{vmatrix} V_s \\ V_i^* \end{vmatrix}$$

Since $I = j\,2\pi f Q$ and $I^* = -j\,2\pi f Q^*$ (5)

we have

$$\begin{vmatrix} I_s \\ I_i^* \end{vmatrix} = \begin{vmatrix} 2\pi f_s C_0 & -2\pi f_s C_1 \\ 2\pi f_i C_1 & 2\pi f_i C_0 \end{vmatrix} \begin{vmatrix} V_s \\ V_i^* \end{vmatrix}$$

Using the matrix representation above we can construct the following four-pole circuit representation.

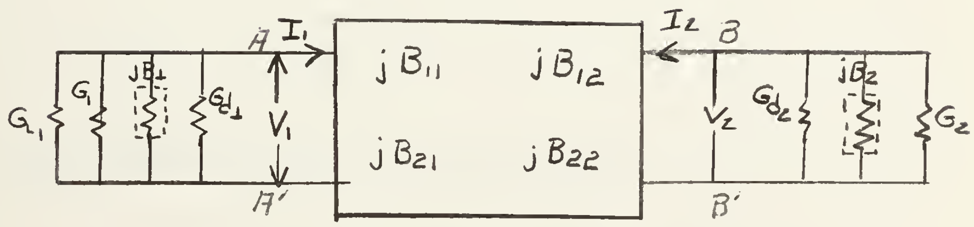


Fig.1

From the above we obtain:

$$I_1 = jB_{11}V_1 + jB_{12}V_2^*$$

$$I_2 = jB_{21}V_1^* + jB_{22}V_2$$

$$I_1/V_1 = jB_{11} \quad + jB_{12}\, V_2^*/V_1$$

$$I_2/V_2 = jB_{21}\, V_1^*/V_2 + jB_{22}$$

$$I_2/V_2 = -Y_2'$$

And

$$V_1^*/V_2 = \frac{-Y_2' - jB_{22}}{jB_{21}}$$

$$V_1/V_2^* = \frac{-Y_2'^* + jB_{22}}{-jB_{21}}$$

the Input Impedance at AA' is expressed as:

$$Y_{in\,AA'} = jB_{11} - \frac{B_{12}B_{21}}{Y_2^* - jB_{22}}$$

As can readily be seen the input impedance presented at plane AA' has a negative real part. Obviously a similar negative conductance results from analyzing the circuit from a corresponding plane BB' indicating the oscillations will occur simultaneously at both idle and signal frequencies.

Representing the total circuit as follows:

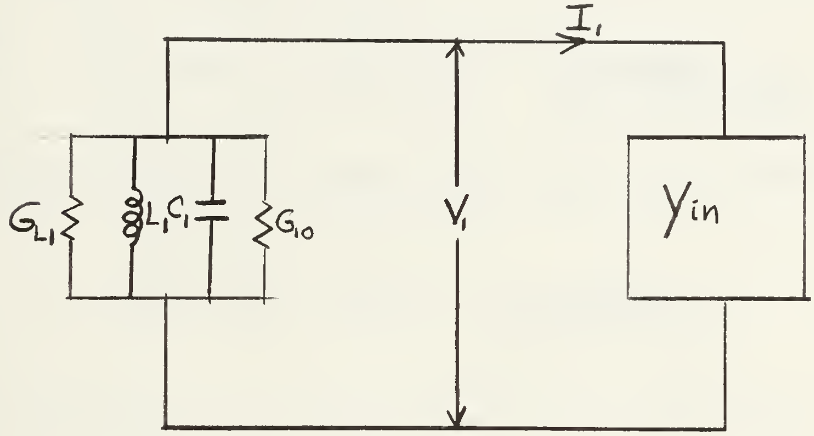


Fig. 2

Where

G_{10} = Effective conductance of tank circuit

C_1 = Effective capacitance of tank circuit

L_1 = Effective inductance of tank circuit

G_L = Effective load conductance

We find that the total admittance seen by circuit one is,

$$Y_{t1} = G_1 + jB_1' + jB_{11} - \frac{B_{12}B_{21}}{Y_2 - jB_{22}}$$

where $G_1 = G_{10} + G_L$,

B_1 = total susceptance of circuit,

and $Y' = G_1 + jB_1'$

Similiarly

$$Y_2' = G_2 + j\,B_2'$$
$$Y_2^* = G_2 - j\,B_2'$$

then $$Y_{t_1} = G_1 + j\,B_1' + j\,B_{11} - \frac{B_{12}\,B_{21}}{G_2 - j\,B_2' - j\,B_{22}}$$

Combining further, the circuit admittance may be expressed as

$$Y_{t_1} = G_1 + j\,B_1 - \frac{B_{12}\,B_{21}}{G_2 - j\,B_2}$$

To cause oscillation the following criteria must be met

$$Yt = 0$$

or $$\mathrm{Re}\left\{Y_t\right\} = 0 = G_1 G_2 + B_1 B_2 - B_{12} B_{21} \qquad (6)$$

$$\mathrm{Im}\left\{Y_t\right\} = 0 = G_2 B_1 - G_1 B_2 \qquad (7)$$

(a) Singled tuned cavity structures:

Assuming both circuits one and two to be operating very near their resonant frequencies ω_{o1} and ω_{o2} respectively, for single tuned cifcuits we may use the following expression for the circuit susceptances:

$$B = \Delta\omega \frac{\partial B}{\partial \omega}$$

In addition, the Q of the circuit can be expressed as

$$Q = \frac{\omega_o}{2G} \frac{\partial B}{\partial \omega}$$

and $$B = \frac{2QG\Delta\omega}{\omega_o} \qquad (8)$$

Thus from equation (7) we have

$$G_2 B_1 = G_1 B_2$$

or $$\frac{B_2}{G_2} = \frac{B_1}{G_1}$$

Substituting from equation (6)

$$\frac{2Q_2\Delta\omega_2}{\omega_{c2}} = \frac{2Q_1\Delta\omega}{\omega_{c1}}$$

where Q_1 and Q_2 equal the loaded Q of the respective circuits;

and $\Delta\omega_n$ equals $\omega_n - \omega_{on}$

and $$\frac{\Delta\omega_1}{\omega_{c1}} = \frac{Q_2}{Q_1}\frac{\Delta\omega_2}{\omega_{c2}} \qquad (9)$$

In order to evaluate the frequency stability of the resulting oscillations it is desired to obtain an expression for the fractional change of the desired signal as related to fractional changes in the frequency of the applied voltage. Recalling the basic frequency relationships of equations (1) we have that

$$f_p = f_s + f_i$$

$$\Delta f_p = \Delta f_s + \Delta f_i$$

Substituting in equation (8) and regarding circuit one as the signal circuit and circuit two as idle circuit we have

$$\frac{\Delta\omega_s}{\omega_s} = \frac{Q_i}{Q_s}\,\frac{\Delta\omega_p - \Delta\omega_s}{\omega_i} ,$$

Finally

$$\delta_{sp} = \frac{Q_i}{Q_s}\,\frac{\omega_p}{\omega_i}\,\frac{1}{1 + \frac{Q_i}{Q_s}\frac{\omega_s}{\omega_i}} \qquad (10)$$

δ sp is defined as the ratio of the fractional detuning of the signal frequency to the fractional detuning of the pump frequency.

$$\frac{\frac{\Delta\omega_s}{\omega_s}}{\frac{\Delta\omega_p}{\omega_p}} = \delta_{sp}$$

Three basic conclusions can be gotten from equation (10)

1. In the degenerate case where ω_s equals ω_i, the stability of the oscillation can be no better than that of the pump frequency.

2. The ratio of pump frequency to signal frequency should be high in order to achieve the greatest improvement in stability.

3. The Q of the signal circuit should be much higher than that of the idle circuit.

Obviously interest lies in making an oscillator as stable as possible; as a result, optimizing the conditions of (2) above it is possible to state the results more simply in the form

$$\delta_{sp} = \frac{Q_i}{Q_s}\,\frac{\omega_p}{\omega_i} \qquad (11)$$

As was pointed out above, to obtain the maximum stability ω_i should approach ω_p In the case necessary for the oscillation to exist, ω_i must always be less than ω_p; so, any improvement in frequency stability must result from the ratio of circuit Q's. In this respect we are fortunate for the following reasons:

At microwave frequencies the cavity Q's are extremely high, 10^3-10^4, therefore, by far the lossiest component of the circuit is the diode. In the current commercial variable capacitor diodes the Q of the diode decreases with increasing frequency for physical reasons, and is at present the limiting factor in the use of the diodes.
In the case of "Varactors" produced by Microwave Associates and used in the experimental tests which follow in Part B the manufacturer states that at any frequency, f, $Q\ max = \frac{f_c}{f}$ where f_c is the cut-off frequency defined as

$$f_c = \frac{1}{2\pi R_s C_{min}}$$

R_s = Equivalent series diode resistance

Therefore, the diode Q's at ω_i and ω_s are inversely proportional to the magnitude of the respective frequencies.

In Appendix I the term filling factor F is derived for a reference between the circuit Q, Q_n, and the diode Q, Q_{Dn}

$$Q_n = \frac{Q_{dn}}{F_n}$$

As a result it is possible to, in general, express equation (10) in terms of the diode Q's:

$$\delta_{sp} = \frac{F_s}{F_i} \frac{Q_{di}}{Q_{ds}} \frac{\omega_p}{\omega_i} \qquad (12)$$

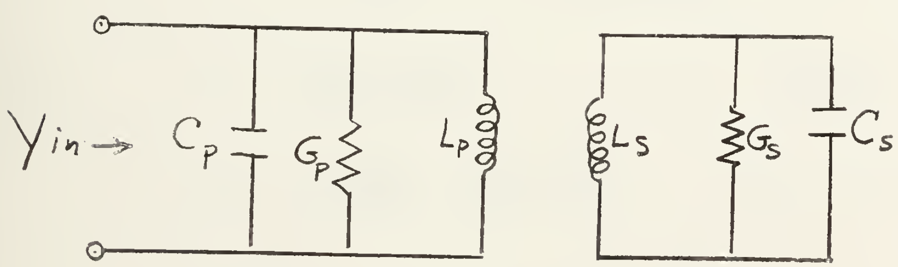
It is now evident that by constructing the circuitry such that there is as much isolation between the circuits as possible, except for the common diode element, and by making the idle frequency cavity constricted and, thereby, causing the filling factor to be as high as possible it should be possible to create a sub-harmonic oscillator that will have a much greater frequency stability than that of the driving or fundamental frequency.

(b) Double tuned circuits:

In the above development we were concerned only with single-tuned circuits for both the idle and signal frequencies. The assumption of single-tuned circuits resulted in the convenient and relatively simple relationships of circuit susceptance, conductance, and Q.

Investigation of the possibility of using double tuned circuits, specifically in the idle circuit results in the following:

Representing the double tuned circuit in lumped circuit constants



Primary Secondary

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We have that

$$y_{in} = \frac{B_p}{Q_p}\left[1 + \frac{K^2 Q_p Q_s}{1+(2\delta_s Q_s)^2} + j2\delta_p Q_p - j\frac{Q_p 2\delta_s Q_s^2 K^2}{1+(2\delta_s Q_s)^2}\right]$$

where $\delta = \frac{\omega - \omega_0}{\omega_0}$ and $\delta_p = \delta_s$

and $\delta << 1$

or, $$y_{in} = \frac{B_p}{Q_p}\left[1 + a + j(2\delta_p Q_p - Q_p b)\right]$$

where $a = \frac{K^2 Q_p Q_s}{1+(2\delta_s Q_s)^2}$ $b = \frac{2\delta_s Q_s^2 K^2}{1+(2\delta_s Q_s)^2}$

$$\frac{B_{in}}{G_{in}} = \frac{2\delta_p Q_p - Q_p b}{1+a}$$

Simplified to a convenient form

$$\frac{B_{in}}{G_{in}} = \frac{A\delta^3 + B\delta}{C\delta^2 + D} = f(\delta)$$

and $A = 8 Q_p Q_s^2$ $C = 4 Q_s^2$

$B = 2Q_p(1 - Q_s^2 K^2)$ $D = 1 + K^2 Q_p Q_s$

Expanded in a Taylor's Series

$$f(\delta) = \frac{B}{D}\delta + \left(\frac{A}{D} - \frac{BC}{D^2}\right)\delta^3 + \left(\frac{BC}{\right.$$

When K = 1/Qs, (critical coupling), B = 0

then

$$f(\delta) = \frac{A}{D}\delta^3 + \frac{AC}{D^2}\delta^5 + \cdots$$

and

$$\frac{B_{in}}{G_{in}} = \frac{8Q_PQ_S^2\delta^3}{1 + Q_S/Q_P}$$

$$= 8Q_LQ_S^2\delta^3$$ to a third order approximation.

where

$$\frac{1}{Q_L} = \frac{1}{Q_P} + \frac{1}{Q_S}$$

This relationship in itself contains too many variables and does not lead directly to any definite conclusion; however, if we can create two identical cavities such that $Q_1 = Q_2 = 1/K$ then the expression for Bin/Gin may be simplified as follows;

$$Bin/Gin = 4Q_P^3\delta^3 \qquad (13)$$

Equation (13) substituted into equation (7) with the idle frequency circuit as the double tuned circuit

$$\frac{2Q_1 \Delta\omega}{\omega_{01}} = 4Q_2^3 \delta_2^3$$

$$2Q_1 \delta_1 = 4Q_2^3 \delta_2^3$$

$$\delta_1 = \frac{2Q_2^3}{Q_1} \delta_2^3 \left(\frac{\omega_3}{\omega_2}\right)^3 \left(1 - \delta_3 \frac{\omega_i}{\omega_3}\right)^3$$

and the fractional detuning ratio of f_1 to f_3 is

$$\delta_{sp} = \frac{2Q_2^3}{Q_1} \left(\frac{\omega_p}{\omega_i}\right)^3 \delta_p^2 \left(1 - \delta_p \frac{\omega_s}{\omega_p}\right)^3$$

with the simplifying assumption that $\delta_p << 1$

and $\omega_i/\omega_3 < 1$ we find that

$$\delta_{sp} \approx 2\, Q_2^3/Q_1 \left(\frac{\omega_p}{\omega_i}\right)^3 \delta_p^2 \tag{14}$$

From the above it is obvious that the stability of the driving force is the important factor in the double tuned case. While the ratio's ω_3/ω_2 and Q_2/Q_1, were the controlling factors in stability control with the singly tuned cavities their roles in the double tuned cavity structure are relatively minor compared to the effect of the stability of the driving force.

From a theoretical standpoint, then, it has been possible to show that oscillators resulting from the negative resistance characteristic of parametric devices can exhibit stability of oscillation greater than that of the driving force. While the basic equations for the device

$$f_p = f_s + f_i$$

$$\text{and} \quad \Delta f_p = \Delta f_s + \Delta f_i$$

can not be violated, $\frac{\Delta f_s}{f_s} \neq \frac{\Delta f_p}{f_p}$ necessarily, but that under controlled conditions the frequency shift may be such that

$$\Delta f_s \to 0$$

$$\text{and} \quad \Delta f_i \to \Delta f_p$$

2. Power Considerations, (Large Signal Analysis):[10]

In the development under small signal analysis in the preceding paragraphs a stability factor was developed for the parametric oscillator. It is admitted, however, that the analysis based solely on small theory makes many assumptions which in the case of infinite gain and the inherent instability which result in oscillation could hardly be classed as small signal theory.

Because of these marginal assumptions in the oscillatory condition it is wise to investigate the device on a limited large signal basis where the combined voltages of the driving frequency, signal frequency and idle frequency are present across the diode.

Also along with the large signal analysis it will be possible to arrive at some power relations between the driving force and the

signal output.

This investigation we will deal primarily with current-voltage relationships in an ideal lumped constant circuit equivalent of the inverting type parametric device employing a nonlinear variable capacitor and the same frequency relationships as in the preceding paragraph. The only assumption on the circuit configuration is that the circuits are of such Q that only voltages near the resonant frequencies of the tank circuits are significant.

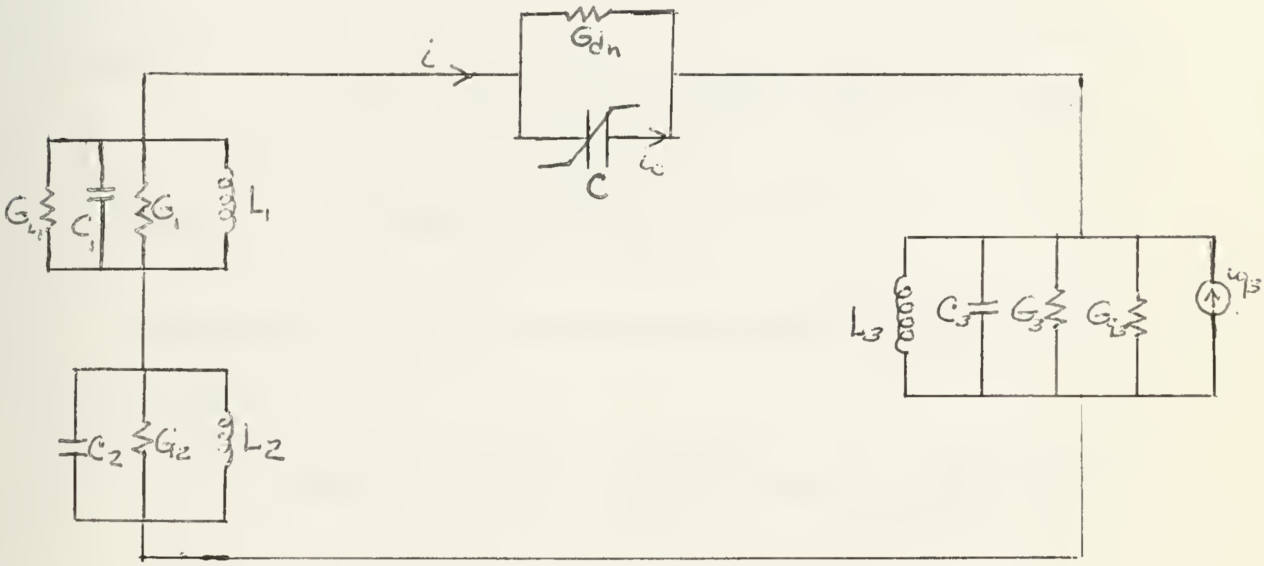

Fig. 3

G_l' = Load conductance

G_g = Internal impedance of pump frequency generator

G_n = Unloaded cavity conductance

L_n = Inductance of cavity n

C_n = Capacitance of cavity n

C = Variable capacitor

G_d = Shunt conductance of nonlinear capacitor

The current i_c across the nonlinear capacitor is described

as
$$i_c = \frac{dq}{dt} = \frac{dq}{dv}\frac{dv}{dt}$$

where
$$\frac{dq}{dv} = C(v)$$

Since in this development we will consider all voltages of frequency f_p, f_s, and f_i as being present across the diode

$$dq/dv = f(v_{ac})$$

where

$$v_{ac} = V_3\cos(\omega_3 t + \Theta_3) + V_2\cos(\omega_2 t + \Theta_2) + V_1\cos\omega_1 t$$

therefore
$$C(v) = C(v_{ac})$$

Expanding $C(V_o)$ in a Taylor's Series about the bias point, V_o, we obtain:

$$C(v_{ac}) = C(V_o) + \frac{\partial C(V_o)}{\partial V} V_{ac} + \frac{1}{2}\frac{\partial^2 C(V_o)}{\partial V^2} V_{ac}^2 \cdots$$

The total current, i, flowing through the capacitor

$$I = G_d V_{ac} + C(v_{ac})\frac{dv_{ac}}{dt}$$

Since the currents are orthogonal functions no mixing of frequencies will be involved and the current may be expressed as the sum of the individual frequency currents

$$i = i(\omega_3) + i(\omega_2) + i(\omega_1)$$

From the Appendix (III) development for $C(v_{ac})\dfrac{dv_{ac}}{dt}$ we have the following for the current components:

$$i(\omega_1) = G_{d1} V_1 \cos\omega_1 t - j\omega_1 C_{bo}\left[1 + \frac{n(n+1)}{2V_0^2}\left(\frac{V_3^2}{2} + \frac{V_2^2}{2} + \frac{V_1^2}{4}\right)\right]$$

$$V_1 \sin\omega_1 t - j\omega_1 \frac{n}{2} C_{bo} \frac{V_2 V_3}{V_0} \sin(\omega_1 t + \Theta_3 - \Theta_2)$$

$$i(\omega_2) = G_{d2} V_2 \cos(\omega_2 t + \Theta_2) - j\omega_2 C_{bo}\left[1 + \frac{n(n+1)}{2V_0^2}\left(\frac{V_3^2}{2} + \frac{V_2^2}{4} + \frac{V_1^2}{2}\right)\right]$$

$$V_2 \sin(\omega_2 t + \Theta_2) - j\omega_2 \frac{n}{2} \frac{C_{bo}}{V_0} V_1 V_3 \sin(\omega_2 t + \Theta_3)$$

$$i(\omega_3) = G_{d3} V_3 \cos(\omega_3 t + \Theta_3) - j\omega_3 C_{bo}\left[1 + \frac{n(n+1)}{2V_0^2}\left(\frac{V_3^2}{4} + \frac{V_2^2}{2} + \frac{V_1^2}{2}\right)\right]$$

$$V_3 \sin(\omega_3 t + \Theta_3) - j\omega_3 \frac{n}{2} \frac{C_{bo}}{V_0} V_2 V_1 \sin(\omega_3 t + \Theta_2)$$

The second term of each expression is due to a linear capacitance across each resonant circuit. The voltage dependence of this capacitance is considered small and the total expression is designated as

$$C' = C_{bo}\left[1 + \frac{n(n+1)}{4V_c^2}\left(V_3^2 + V_2^2 + \frac{V_1^2}{2}\right)\right]$$

The remainder of the terms in the circuit equation, are linear once we disregard the voltage squared term in C'.

Letting the term $n\frac{C_{bo}}{V_o} = K$ for convenience, we can write the circuit admittances as follows

$$Y_1 = G_{d_1} + j\omega_1 C' + j\omega_1 \frac{K V_2 V_3}{2V_1} e^{j(\Theta_3 - \Theta_2)}$$

$$Y_2 = G_{d_2} + j\omega_2 C' + j\omega_2 K \frac{V_1 V_3}{2V_1} e^{j(\Theta_3 - \Theta_2)}$$

$$Y_3 = G_{d_3} + j\omega_3 C' + j\omega_3 K \frac{V_2 V_1}{2V_3} e^{j(\Theta_2 - \Theta_3)}$$

The circuit equations then follow as:

$$0 = V_1 Y_{T_1} + j\frac{\omega_1 V_2 V_3 K}{2} e^{j(\Theta_3 - \Theta_2)} \tag{15}$$

$$0 = V_2 Y_{T_2} + j\,\omega_2 \frac{V_1 V_3}{2} K e^{j(\Theta_3 - \Theta_2)} \tag{16}$$

$$I_{g3} = V_3 Y_{T3} + j\,\omega_3 \frac{V_1 V_2}{2} K e^{j(\Theta_2 - \Theta_3)} \tag{17}$$

where

$$Y_{tn} = G_{Tn} + j\left(\omega_n C_n + C' - \frac{1}{\omega_n L_n}\right)$$

and

$$G_{T1} = G_1 + G_{d1} + G_L$$

$$G_{T2} = G_2 + G_{d2}$$

$$G_{T3} = G_{g3} + G_3 + G_{d3}$$

Solving for V_2 and taking the complex conjugate we have:

$$V_2 = j\,\frac{\omega_2 K V_1 V_3}{2 Y_{T2}^{*}}\, e^{-j(\theta_3 - \theta_2)}$$

After eliminating V_2 from the remaining equations we have

$$0 = V_1 Y_{T1} - \frac{\omega_1 \omega_2 K^2 V_1 V_3^2}{4 Y_{T2}^{*}}$$

and

$$I_{g3} = V_3 Y_{T3} + \frac{\omega_2 \omega_3 K^2 V_3 V_1^2}{4 Y_{T2}}$$

The negative conductance characteristic of the device is apparent at f_1, and if V_1 had been eliminated in the set of current equations rather than V_2, the negative conductance characteristic would have been evident at f_2 also leading to the conclusion stated that oscillation will occur simultaneously at both signal and idle frequencies.

Obviously in considering oscillation produced by this device we must consider the circuit at or very near to resonance so that the above relations can be written as

$$0 = V_1 G_{T_1} - \frac{\omega_1 \omega_2 K^2 V_1 V_3^2}{4 G_{T_2}}$$

and

$$I_{g3} = V_3 G_{T3} + \frac{\omega_2 \omega_3 K^2 V_3 V_1^2}{4 G_{T2}}$$

and the circuit conductances

$$G(\omega_1) = G_{T_1} - \frac{\omega_1 \omega_2 K^2 V_3^2}{4 G_{T2}}$$

$$G(\omega_3) = G_{T3} + \frac{\omega_2 \omega_3 K^2 V_1^2}{4 G_{T2}}$$

To commence oscillation

$$G(\omega_1) = 0$$

or

$$G_{T_1} = \frac{\omega_1 \omega_2 K^2 V_3^2}{4 G_{T2}} \qquad (18)$$

Solving for $V_1 K$ and substituting in the expression for the driving circuit admittance we have that

$$G(\omega_3) = \frac{I_{g3}}{2} \sqrt{\frac{\omega_1 \omega_2 K^2}{G_{T_1} G_{T2}}}$$

since

$$G(\omega_3) = \frac{I_{g3}}{V_3}$$

$$V_3^2 = \frac{4 G_{T_1} G_{T_2}}{\omega_1 \omega_2 K^2}$$

Practically speaking this is the value of pumping voltage across the diode required to commence oscillation:

The power required, across the diode, to start oscillation is therefore

$$P(osc) = \frac{V_3^2 G_{d_3}}{2} = \frac{2 G_{T_1} G_{T_2} G_{d_3}}{\omega_1 \omega_2 K^2}$$

and

$$C_{d_3} = \frac{\omega_3 C_{bo}}{Q_{d_3}}$$

therefore

$$P(osc) = \frac{2 G_{T_1} G_{T_2} \omega_3 C_{bo}}{\omega_1 \omega_2 K^2 Q_{d_3}} \tag{19}$$

If we assume that for microwave circuits that G_{d_n} is $<< G_n$ at all frequencies then equation (19) may be written as

$$P(osc) = \frac{2 G_{T_1} G_{T_2} C_1 C_2 \omega_3 C_o}{\omega_1 C_1 \omega_2 C_2 K_1^2 Q_{d3}} = \frac{2 \omega_3 C_{bo} C_{10} C_{20}}{Q_1 Q_2 K^2 Q_{d3}}$$

Where C_{10} and C_{20} are the total circuit capacitances.

At the idle frequency there is no external load so that Q_2 may be expressed as

$$\frac{1}{Q_2} = \frac{1}{Q_{c2}} + \frac{1}{Q_{d_2}'} = \frac{F_2}{Q_{d2}}$$

Where now Q_{c2} is the unloaded cavity Q and is much greater than Q_d. In the signal circuit there is, however, a necessary external load to couple out the signal.

The total Q_1 is the loaded Q of this circuit such that

$$1/Q_1 = \frac{1}{Q_{c_1}} + \frac{1}{Q_d''} + \frac{1}{Q_{ext}}$$

Again assuming that the cavity losses are negligible to the diode losses we have

$$1/Q_1 = \frac{F_1}{Q_d} + \frac{F_1 \beta}{Q_d}$$

where β is the coupling coefficient of the signal circuit.

or
$$1/Q_1 = \frac{F_1}{Q_d}(1+\beta)$$

Then the pumping power required to commence oscillation can be expressed as

$$P_{(osc)} = \frac{2\omega_3 C_{bi} F_1 F_2 C_1 C_2 (1+\beta)}{Q_{d_1} Q_{d2} Q_{d3} K^2}$$

Applying the relationship of the filling ractor once again

$$F_n = \frac{C_{dn}}{C_{dn} + C_n}$$

we have that

$$P_{(osc)} = 2\left(\frac{C_{bo}}{K}\right)^2 \frac{\omega_3 C_{bo}(1+\beta)}{Q_{d_1} Q_{d2} Q_{d3}} \tag{20}$$

All factors in the above equation can be evaluated or closely approximated by manufacturer's data so that an approximate evaluation of the pumping power required can be made. Evaluation of this required pumping power is made in the sector devoted to experimental results.

In considering the power output of the oscillator at the desired frequency we realize that oscillations should occur at a particular value of pumping power and build up to a certain level determined by the characteristics of the device. That there is a limiting level is evident from the equation for the admittance of the pump circuit. It is clear that the admittance of this circuit increases with the signal output and with its own increasing voltage drive. We can conclude then that the output oscillation will be limited finally at some point where any increase of driving power is dissipated in the pumping circuit itself rather than in contributing to the oscillatory output.

We can expect, therefore, that a plot of output voltage versus pumping power will commence abruptly at a point $P(osc)$ and increase to a limiting value where any additional increase in pumping power is dissipated in the circuitry itself, and then will decrease gradually to a value where the oscillations again cease.

Power output:

Signal power output can be expressed as

$$P_1 = \frac{V_1^2}{2} G_{L_1}$$

and at resonance

$$G_{\omega_3} = G_{T3} + \frac{\omega_2 \omega_3 (K V_1)^2}{4 Y_{T2}}$$

$$G_{\omega_3} = G_{T3} + \frac{2 \omega_2 \omega_3 K^2 P_1}{4 G_{T2} G_{L1}}$$

therefore

$$P_1 = \frac{2 G_{T1} G_{L1} (G_{\omega 3} - G_{T3})}{\omega_2 \omega_3 K^2}$$

and

$$P_{3(osc)} = \frac{2 G_{T1} G_{T2} G_{d3}}{\omega_1 \omega_2 K^2}$$

At the point of oscillation we have power to cause oscillation; at a point slightly beyond this point

$$\eta = \frac{P_1}{P_{osc}} = \frac{\omega_1}{\omega_3} \frac{G_{L1}}{G_{T1}} \left[\frac{G_{\omega 3} - G_{T3}}{G_{d3}} \right]$$

At a point just prior to oscillation

$$G_{\omega 3} = G_{T3}$$

and the efficiency is zero As the device oscillates the second term of the expression for $G_{\omega 3}$ becomes larger and the difference $G_{\omega 3} - G_{T3}$ becomes finite As pointed out previously there soon becomes a point when the pumping power involved in sustaining oscillation is no longer dissipated across the diode and the expression for $P_{(osc)}$ no longer holds

In the limiting case one might conclude that with assumed lossless diodes the efficiency of the diode would approach that of the Manley-Rowe relations[5].

$$\eta_{max} = \omega_1/\omega_3$$

Up to this point we have ignored the second term in the expression for C'. This term as derived is a function of the square of the voltages. In the above analysis this term was ignored because in development of the series representation for $C(v)$ was assumed that $V_3 + V_2 + V_1 < V_0$. From Appendix III we found that the term $\frac{\partial^2 C}{\partial V_0}(V_0)$ contained a term inversely proportional to V_0^2. In the original assumption of disregarding the voltage square term of C' it was realized that if $V_3 + V_2 + V_1 < V_0$ that $V_3^2 + V_2^2 + V_1^2$ would be smaller than V_0^2. While this assumption was valid in the non-oscillatory condition because V_1 and V_2 were Zero until oscillation occurred, it can not be ignored completely once oscillation occurs. The build up of V_1 and V_2 when oscillation occurs plus the increase in V_3 as $P(osc)$ is exceeded may well cause the term to be quite significant. The presence of this term will cause a detuning affect on the circuits. If this term is large enough to detune the circuits it is conceivable that this effect could more than cancel the desired frequency stability relations obtained in Part A. Since this term is an effective additive capacitance the presence of this detuning phenomenon should indicate its presence as a decrease in the frequency of the output signal as the amplitude of oscillation increases caused by an increase in pumping power.

From the foregoing large signal analysis we can therefore draw the following conclusions:

(1) Oscillation is possible.

(2) A finite amount of power is required at the pumping frequency to cause oscillation.

(3) The efficiency of the device will approach ω_s/ω_p as an upper-limit.

(4) Increase in input power cause a corresponding increase in the circuit admittance and therefore limit the amplitude of out put voltage.

(5) The circuit may experience a detuning effect at large voltage amplitudes of both input and output voltage.

B. Experimental Results

1. Oscillator design

The device constructed for investigation of the theory put forth in the preceding paragraphs was a single tuned cavity structure both for the idle and signal frequencies. Single tuned cavities were preferred because of their simplicity in construction in the time available.

In order to take the maximum advantage of the frequency relationships that enter the stability factor the original design frequencies were: signal frequency; 1000mc; idle frequency; 9000mc; pump frequency; 10kmc. Subsequent destruction of the original diode and frequency response of the radial choke necessitated changing these frequencies to: 900mc, 8100mc and 9000mc, respectively. These frequencies allow a favourable relationship for the diode Q's and the ratio of pump f frequency to idle frequency.

The variable reactance device used for the parametric effect was a Microwave Associates "Varactor" type MA460C with a cutoff frequency, fc, of 50KMC.

The signal cavity is a resonant coaxial cavity with a characteristic impedance of 51 ohms. Signal output is obtained by an inductive loop to a standard "N" type fitting.

In order to obtain a reasonable amount of isolation in the output from the idle and driving frequencies a radial choke was constructed in the coaxial cavity to prevent the pump frequency from propogating in the signal cavity.

The diode is mounted across a standard X-band wave guide. Coupling to the signal cavity is achieved by inserting the nipple of the diode

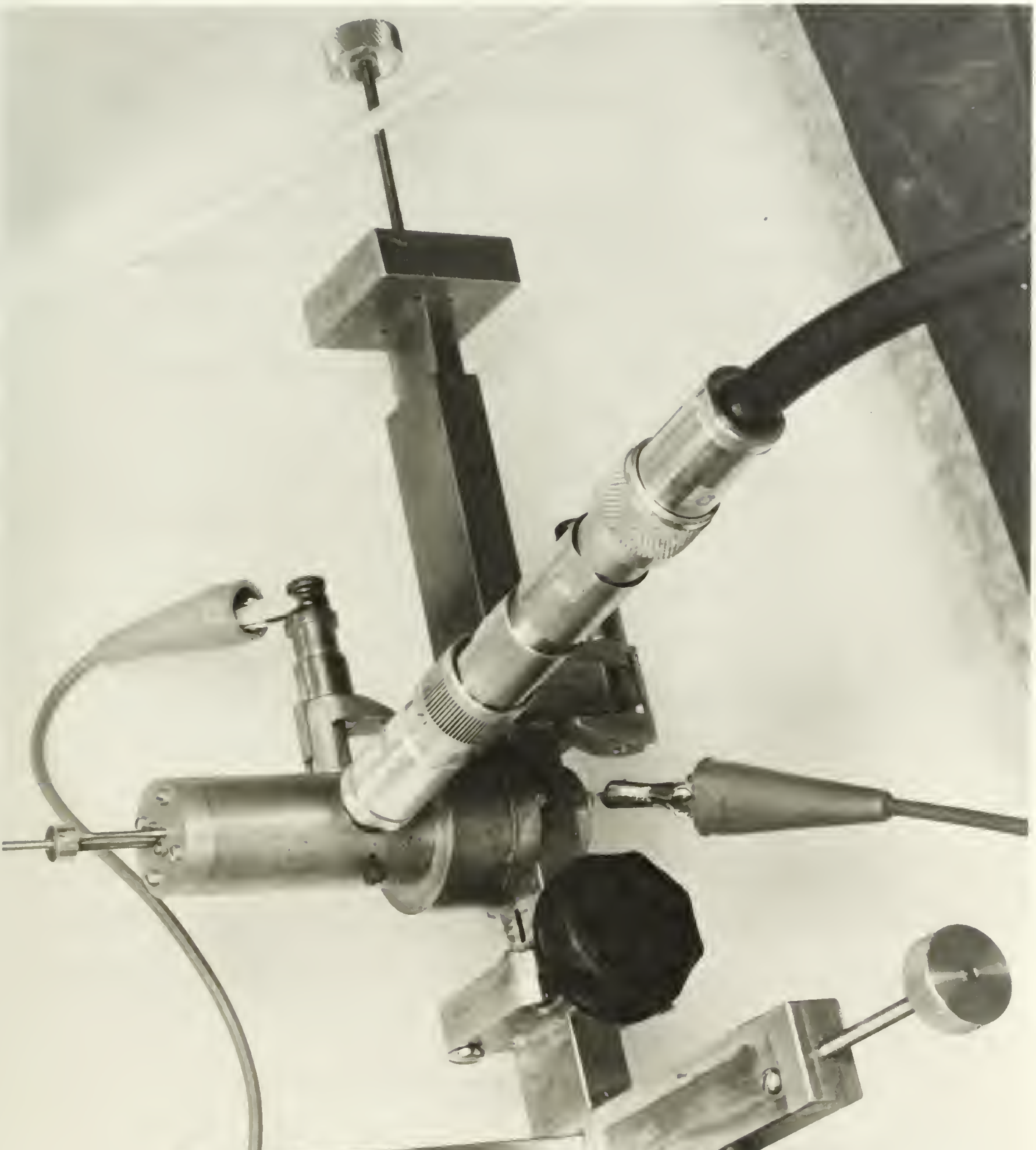

Fig. 4

in the coaxial center conductor The base of the diode is isolated from the wave guide by a mica strip which allows a bias voltage to be applied to the diode.

Both the idle cavity and pump cavity were allowed to propogate in waveguide. The idle cavity was formed by a variable short at the extreme end and a slide screw tuner on the power input side. The idle cavity was tuned by setting the short so that a voltage maximum at the idle frequency would appear across the diode and then the tuner adjusted so that the line was flat at the idle frequency.

The pump frequency circuit was then formed in identically the same manner by using a variable short at the extreme terminal that was concentric with that used for the idle frequency but with a cutoff frequency of 8.4KMC. The input side was then tuned by use of an E-H tuner of the same cutoff frequency. Investigation of the effectiveness of these tuners indicated that they introduced no effect at the idle frequency.

The pumping power was supplied by a Varian Associates klystron VA 58. Figure 4 is a photograph of the completed device.

2. Preliminary evaluations

In order to substantiate theory it was necessary to evaluate the circuit filling factors and circuit Q's at the respective frequencies. Appendix III contains data on the diode capacitance versus bias voltage, the evaluation of the exponential, n, the circuit Q's and the variation of circuit resonance with diode bias voltage.

From the graphical data it was possible to evaluate the following parameters:

V_b	2 volts
ϕ	0.25 volts
Q_1	60
Q_2	34.7
F_1	0.43
F_2	0.0675
n	0.223
C_{bo}	1.68 uuf
K	0.1665 uuf/volt

The filling factor was evaluated as follows:

At resonance

$$\omega_0^2 = \frac{1}{L_n C_n}$$

$$(\omega_0 + \Delta\omega)^2 = \frac{1}{L_n (C_n + \Delta C)}$$

Assuming that $\Delta\omega$ is very small

$$\frac{\Delta\omega}{\omega_0} = -\frac{\Delta C}{2C}$$

and

$$\frac{\Delta\omega}{\omega_0} = \frac{\Delta C}{2(C + C_{bo})}$$

Therefore

$$F = \frac{\Delta f / f_0}{\frac{1}{2} \frac{\Delta C}{C_{bo}}}$$

In relations where the diode Q's are applicable the manufacturers evaluation is considered as satisfactory. Any discrepancy here should be proportional with frequency and should not affect the expected

results. Consequently the diode Q's are approximated as follows:

Q_1	@ 870mc	57.5
Q_2	@ 8120mc	6.15
Q_3	@ 9000 mc	5.55

With the above data available we can evaluate the theoretical relations made in Part A.

Stability of oscillation.

$$\delta_{sp} \simeq \frac{\omega_3}{\omega_2} \frac{Q_2}{Q_1} \tag{11}$$

Power to commence oscillation.

$$P_{osc} = 2\left(\frac{C_{bo}}{K}\right)^2 \frac{\omega_3 C_{bo}}{Q_{d1} Q_{d2} Q_{d3}} \tag{20}$$

Substitution of the experimentally derived from "a priori" data in equations (11) and (20) give for expected values,

$$\delta_{sp} = 0.634$$

$$P_{osc} \simeq 34 \text{ mw}$$

3. Experimental Results

As explained in paragraph (1) above both the idle and signal frequency cavities were tuned to their respective frequencies and an X-band source at 9.0 Kmc used as the pump frequency. A finite amount of final tuning of the signal cavity was required subsequent to application of the driving power in order to obtain the exact resonant frequency responses. The final frequency values were:

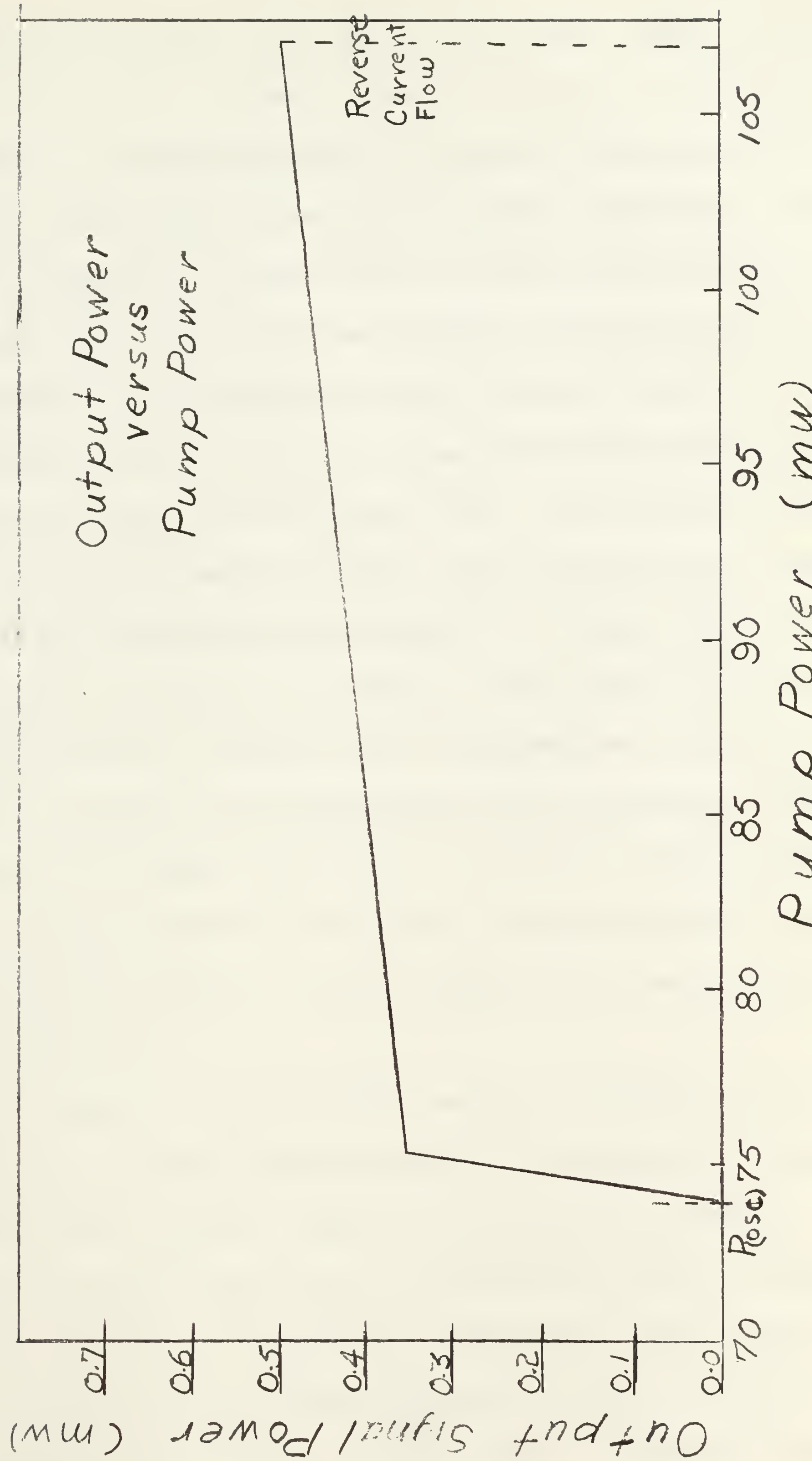
Output Power
versus
Pump Power
Reverse
Current
Flow
Pump Power (mw)
70
75
80
85
90
95
100
105
Posc
Output Signal Power (mw)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

Fig. 5

Initial oscillation occurred at a pump power value of 74 mw vs the estimated 34mw. The output power rose sharply at this point as expected and increased uniformly to a value of one hundred twenty-eight per cent of its initial value. At the maximum output value there was noted an indication of reverse diode current and beyond this point the output power began to decrease rapidly. The peak value of output was reached at a driving power value of 106.7mw. From these results it was noted that the increase in output power was proportional to the square root of the increase in power input. This agrees with the relation derived by Kutzebue between pump power and output power. Figure (5) is a graphical presentation of the output power versus input power over the range where no reverse current flowed in the diode.

Since the basic concern of this experiment was with frequency stability no attempt was made to evaluate the efficiency of the energy conversion. The desired high Q of the signal circuit necessitated that the output coupling of the signal cavity be as small as possible and still allow detection. For this reason no attempt was made to attain maximum efficiency.

The frequency stability of the output in relation to the input was made in a rather rudimentary fashion by varying the reflector voltage of the Klystrom. Frequency measurements were made by use of a Hewlett Packard Transfer Oscillator Model 540A used in conjunction with a Hewlett Packard Frequency Counter Model FR38U. Measurements of frequency were made at an arbitrary value of input power midway between that which caused oscillation and that value at which the output power was a maximum. The results of these measurements are tabulated in

Figure (6). The measurements indicate that the frequency stability is far from improved by use of the parametric device. In the best case δ_{sp} was able to approach 5.0.

Subsequent observations led to the results illustrated in Figure (7) in which we can observe the detuning effect described in the theoretical considerations under the large signal analysis. The effects are quite evident and in part explain the frequency stability results obtained previously. As predicted the application of increased pumping power lowered the output frequency. The effect appears to be an exponential decrease, leveling off at the value of output frequency at which the corresponding value of pumping power causes a flow of reverse diode current. In regard to the results obtained from stability considerations we note that the output frequency increased with a decrease in pumping frequency. This can in part be explained by the fact that variations of the repellor voltage cause a detuning of the Klystron and in addition to effecting slight changes in the frequency also reduce the power output of the Klystron provided the Klystron was originally properly tuned. This decrease in input power, according to the results illustrated in Figure (7), should result in an increase in output frequency, a hypothesis born out by the results of the frequency measurements since the Klystron was at its optimum tuned condition at an output frequency of 9000.0 Kmc. This apparent explanation of the results obtained leads to the supposition that the stability theory put forth in Part A may not be wholly discredited by this experiment but under more rigid controls could be substantiated.

Pump Frequency (MC)	Signal Frequency (MC)	Δf_p	Δf_s	δ_{sp}
9000.95	856.23			
8999.55	857.15	1.40	0.92	7.0
8999.14	857.52	1.81	1.29	7.53
8997.32	858.15	3.63	1.92	5.5
8994.81	859.5	6.14	3.27	5.6

Fig 6

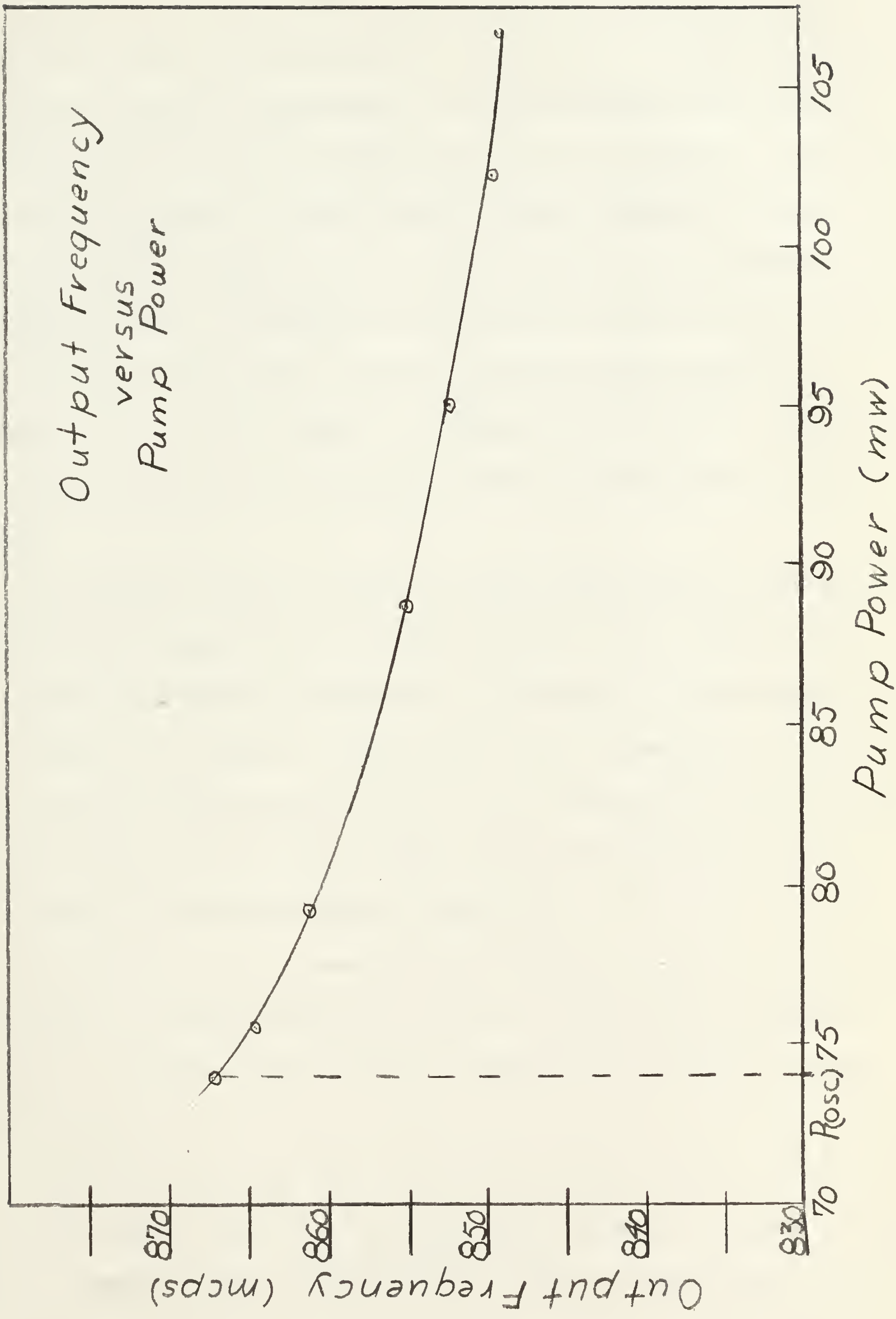
Output Frequency
versus
Pump Power
Pump Power (mw)
Output Frequency (mcps)
70
75
80
85
90
95
100
105
Posc
830
840
850
860
870

Fig. 7

C. Conclusions:

The results of this experiment are considered inconclusive in regard to proving or disproving the derived theory of stability in output oscillation of the negative resistance parametric oscillator. Proved conclusively were the propositions that the device is capable of oscillation and that the device suffers from an inherent detuning affect with variations of pumping power. With these facts now established it is believed that a new approach to proving the stability theory can be made with the object being to obtain a driving source which can be minutely tuned and still maintain a constant output in order that frequency measurements can be obtained without introducing the detuning affect.

It is possible that in order to receive the greatest advantage of the stability relationship one should design a device such that the idle cavity is a doubly tuned circuit. One method of accomplishing this would be to design the idle circuit that is composed of two identical cavities each containing one of a set of matched diodes to effect the identical characteristics needed to take full advantage of the conductance- usceptance characteristics.

In addition it is recommended that a greater degree of isolation be accomplished among the three circuits in order that any required frequency adjustments may be effected completely independent of the other circuits.

However, despite any of the above improvements in the device itself the obvious presence of the detuning effect and its affect on the output frequency leads to the conclusion that use of the device for a stable frequency source is going to be quite limited. Variations in

pump frequency may be tolerated but at this stage it appears that any frequency variations must be accompanied by a constant power output despite the variations of frequency. Under these conditions proof of the theory set forth in this paper may be significant and further investigation warranted.

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APPENDIX I

FILLING FACTOR

The term filling factor, F, is a convenient way of relating the energy storage capacity of the diode to the energy stored in the entire circuit under consideration. Filling factor is defined as follows.

$$F_n = \frac{\text{Energy stored in the diode at frequency } f_n}{\text{Total energy stored in the circuit at frequency } f_n}$$

or

$$F_n = \frac{\text{Diode capacitance } (C_{dn})}{\text{Total circuit capacitance } (C_n + C_{dn})} \qquad (1)$$

The quality factor, Q, of a circuit is a measure of the energy storage capability of the circuit and is expressed as

$$Q = \omega_n \frac{(\text{total energy stored})}{\text{power loss}}$$

The Q of circuit n.

$$1/Q_n = \frac{1}{Q} + \frac{1}{Q_D}$$

$$1/Q_n = \frac{G_n}{\omega(C_n + C_{dn})} + \frac{G_d}{\omega(C_n + C_{dn})}$$

From (1) above

$$1/Q_n = \frac{FG_n}{\omega C_{dn}} + \frac{FG_d}{\omega C_{dn}}$$

Assuming that the losses in the diode are much greater than those of the cavity we have that

$$Q_n = \frac{Q_{dn}}{F_n} \qquad (2)$$

Thus from the filling factor we are able to evaluate the Q of the circuit in terms of the diode at a particular frequency. Since the Q of the diode is very frequency dependent the ability to express the Q of the circuit in terms of the Q of the diode is relatively important.

APPENDIX II

Signal Analysis of $C_{(v)}$

For the diodes used in this analysis C is expressed in the following manner

$$C = \frac{C_o}{\left(1 + \frac{V_b}{\phi}\right)^n}$$

C_o = Value of the diode capacitance at zero volts bias

ϕ = Diffusion junction voltage for silicon graded junction diodes such as the Microwave Associates diode as used in this experiment.

n = Is specified as between 0.5 and 0.333 by the manufacturer

V_b = Absolute value of bias voltage

To express the value of capacitance variation about a particular bias voltage V_b caused by a periodically varying voltage the following expression is used.

$$C(v) = \frac{C_{bo}}{\left(1 + \frac{V_t}{V_o}\right)^n}$$

where

$C_{bo(t)}$ = Diode capacitance at bias

V_o = $\phi + V_b$

$V(t)$ = Time varying impressed voltage on diode

For the small signal analysis where the signal voltage and idle voltage are considered very much smaller than the driving voltage V_3

$$C(v) = \frac{C_{bo}}{\left(1 + \frac{v_0}{V_0}\right)^n}$$

and

$$v(t) = V_3 \cos \omega_3 t$$

The series expansion of $C_{bo(t)}$ about V_o is expressed in the following manner

$$C(t) = C_{bo}\left[1 - n\frac{v_3}{v_0} + \frac{n(n+1)}{2}\left(\frac{v_3}{V_0}\right)^2 - \frac{n(n+1)(n+2)}{3!}\left(\frac{v_3}{V_0}\right)^3 + \frac{n(n+1)(n+2)(n+3)}{4!}\left(\frac{v_3}{V_0}\right)^4 + \text{------}\right]$$

$$C(t) = C_{bo}\left[1 - \frac{n}{V_0} V_3 \cos \omega_3 t + \frac{n(n+1)}{2V_0^2} V_3^2 \cos^2 \omega_3 t - \frac{n(n+1)(n+2)}{3!} V_3^3 \cos^3 \omega_3 t + \text{----}\right]$$

In the analysis of the parametric device it is assumed that only the fundamental component of the frequency is allowed to propagate, hence

$$C(v) = C_{bo}\left[1 - \frac{n}{V_0} V_3 \cos \omega_3 t + \frac{n(n+1)}{2V_0^2}\frac{V_3^2}{2} - \frac{n(n+1)(n+2)3}{6V_0^3 4}\right.$$

$$V_3^3 \cos \omega_3 t + \frac{n(n+1)(n+2)(n+3)3}{24 V_0^4\, 4} V_3^4$$

$$C_{(v)} = C_{b0} \left\{ 1 + \frac{n(n+1)}{4 V_0^2} V_3^2 - \left[\frac{n}{2} \frac{V_3}{V_0} + \frac{n(n+1)(n+2)}{16 V_0^3} V_3^3 \right] \cos \omega_3 t \right\}$$

$$C_{(v)} = C_{b0} K - C_{b0} K_1 \cos \omega_3 t$$

$$= C_0 + C_1 (v_3)$$

where

$$K_0 = 1 + \frac{n(n+1)}{4 V_0^2} V_3^2$$

$$K_1 = \frac{n}{2} \frac{V_3}{V_0} \left[1 + \frac{(n+1)(n+2)}{8} \left(\frac{V_3}{V_0} \right)^2 \right]$$

APPENDIX III

LARGE SIGNAL ANALYSIS OF $C_{(v)}$

In the following large signal analysis the magnitudes of the signal and idle voltages and V_1 and V_2, are not considered much smaller than V_3; therefore the capacitance variation of the diode must be a function of all three voltages V_3, V_2, V_1.

$$C(v) = \frac{C_{bo}}{\left(1 + \frac{V_t}{V_o}\right)^n}$$

C_{bo} = Defined as in Appendix II.

V_o = Defined as in Appendix II.

V_t = $V_3 \cos(\omega_3 t + \Theta_3) + V_2 \cos(\omega_2 t + \Theta_2) + V_1 \cos \omega_1 t$

Θ_2 and Θ_3 are arbitrary phase angles associated with V_2 and V_3 respectively.

Expanded about the bias voltage V_o the capacitance can be expressed in the following manner.

$$C(v) = C_{bo}(V_o) + \frac{\partial C_{bo}}{\partial V}(V_o)\, V(t) + \tfrac{1}{2}\, \frac{\partial^2 C}{\partial V^2}(V_o)\, V_t^2$$

$$+ \tfrac{1}{6}\, \frac{\partial^3 C_{bo}}{\partial V^3}(V_o)\, V_t^3 + - - - - - -$$

$$C\, v = C_{bo} \Big[1 - \frac{n}{V_o} V_t + \frac{n(n+1)}{2 V_o^2} V_t^2$$

$$- \tfrac{1}{6}\, \frac{n(n+1)(n+2)}{V_o^3}\, V_t^3 + - - - - -$$

For comparative ease of computation only the terms of C through the second (2nd) order will be used

$$C(t) = C_{bo}\left[1 - \frac{n}{V_c} V(t) + \frac{n(n+1)}{2 V_c^2} V^2(t)\right]$$

This expansion is the same as that obtained in Appendix I. In this case, however, the idle frequency voltage and the signal voltage in addition to the driving voltage will flow across the diode.

Further development of the above expressions into the relation for current,

$$i_c = C(t) \frac{dv}{dt}$$

and including only those components of frequency f_1, f_2, and f_3 we have that

$$i_c = i_{c_1} + i_{c_2} + i_{c_3}$$

$$i_{c_1} = j\omega_1 C_{bo}\left[1 + \frac{n(n+1)}{2 V_c^2}\left(\frac{V_3^2}{2} + \frac{V_2^2}{2} + \frac{V_1^2}{4}\right)\right] V_1 \sin \omega_1 t$$

$$- j\omega_1 \frac{n}{2} \frac{V_2 V_3}{V_c} \sin(\omega_1 t + \Theta_3 - \Theta_2)$$

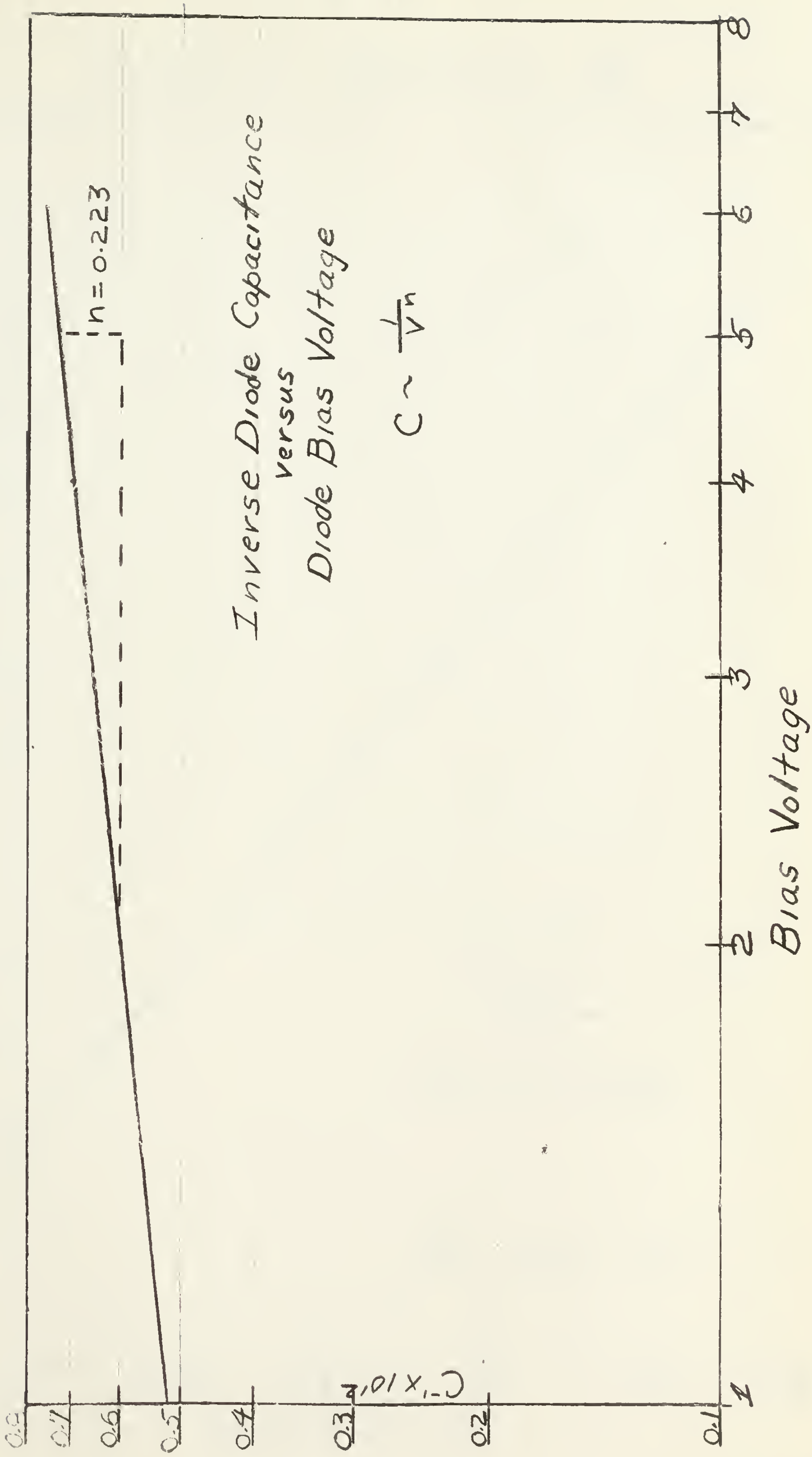
$$i_{c_2} = j\omega_2 C_{bo}\left[1 + \frac{n(n+1)}{2 V_c^2}\left(\frac{V_3^2}{2} + \frac{V_2^2}{4} + \frac{V_1^2}{2}\right)\right] V_2 \sin(\omega_2 t + \Theta_2)$$

$$- j\omega_2 \frac{n}{2} \frac{V_1 V_3}{V_c} \sin(\omega_2 t + \Theta_3)$$

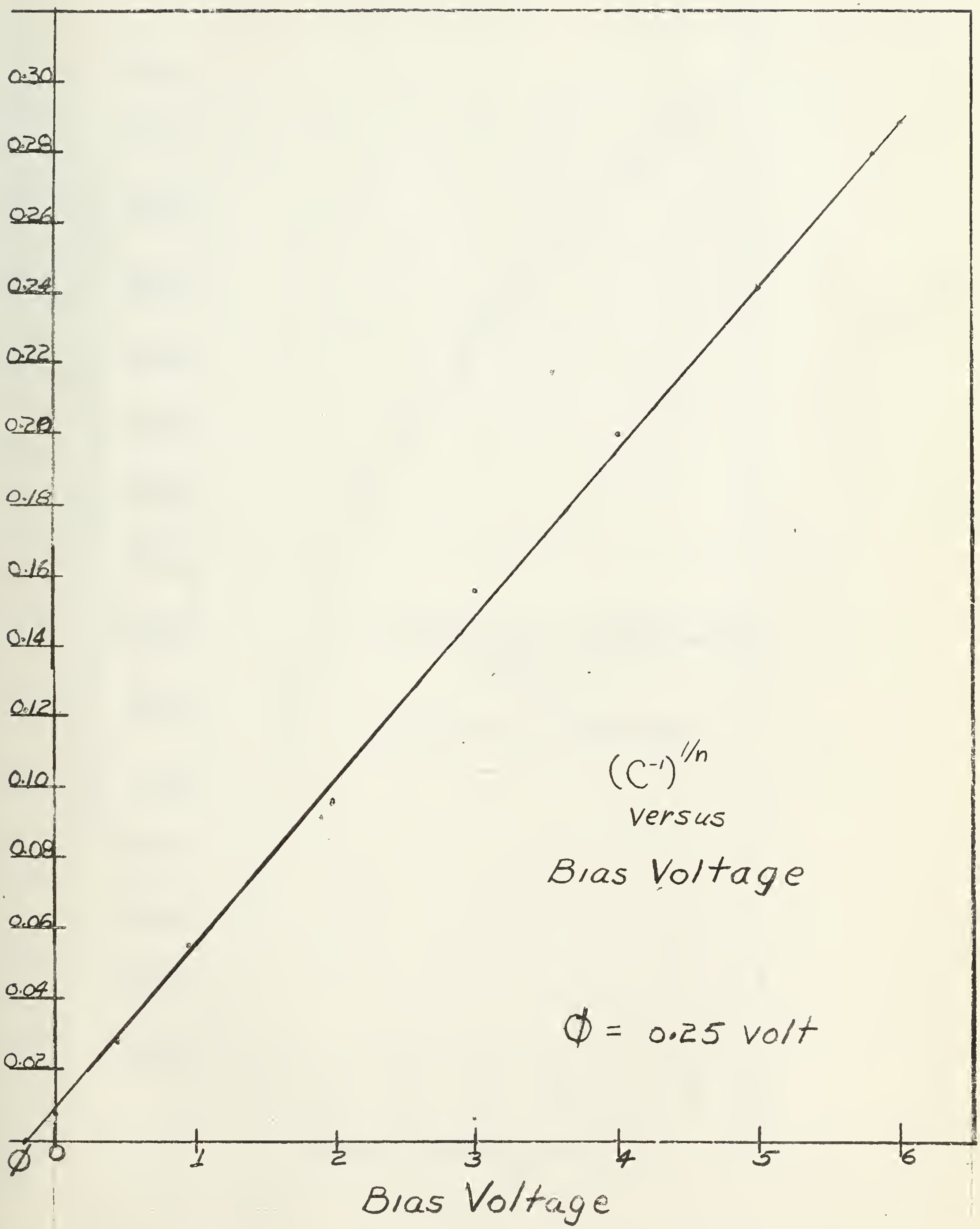
$$i_{c_3} = j\omega_3 C_{bo}\left[1 + \frac{n(n+1)}{2 V_c^2}\left(\frac{V_3^2}{4} + \frac{V_2^2}{2} + \frac{V_1^2}{2}\right)\right] V_3 \sin(\omega_3 t + \Theta_3)$$

$$- j\omega_3 n/2 \frac{V_1 V_2}{V_c} \sin(\omega_3 t + \Theta_2)$$

APPENDIX IV

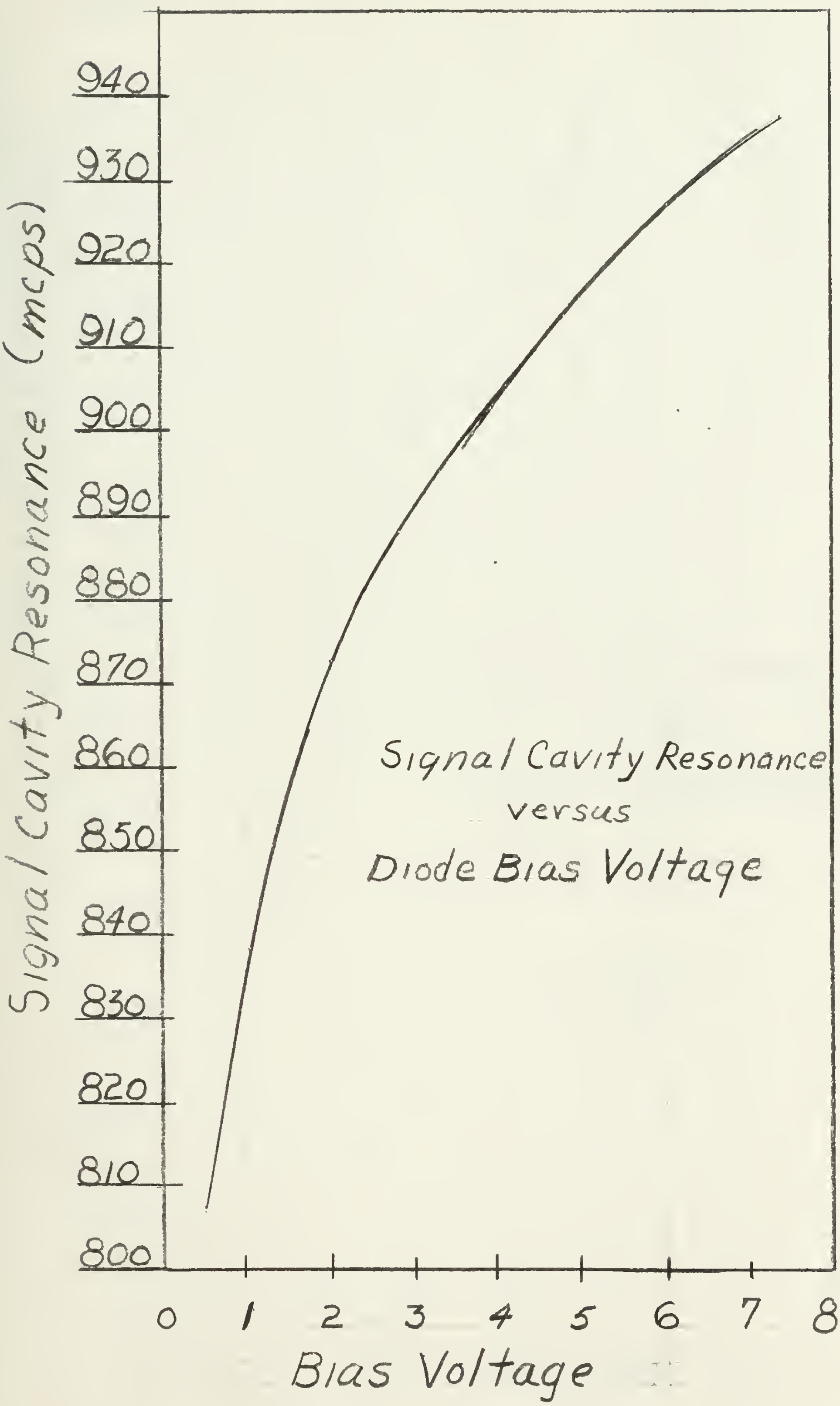

Inverse Diode Capacitance
versus
Diode Bias Voltage
C ~ 1/V^n
n = 0.223
Bias Voltage
C^-1 x 10^12
1
2
3
4
5
6
7
8
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

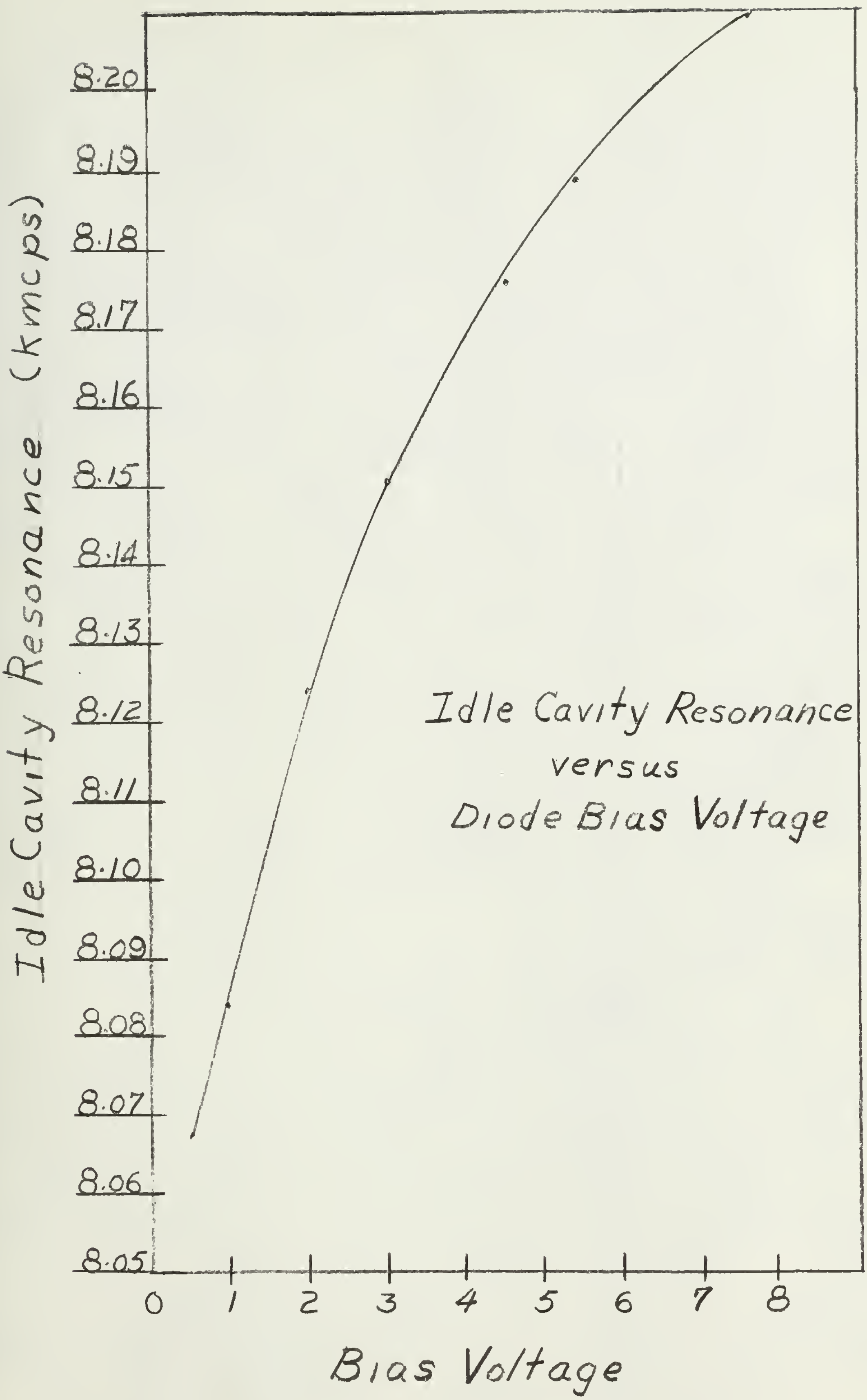
Difussion Potential, ϕ



Bias Voltage

APPENDIX V


Signal Cavity Resonance (mcps)
940
930
920
910
900
890
880
870
860
850
840
830
820
810
800
0
1
2
3
4
5
6
7
8
Bias Voltage
Signal Cavity Resonance
versus
Diode Bias Voltage


Idle Cavity Resonance (kmcps)
8.20
8.19
8.18
8.17
8.16
8.15
8.14
8.13
8.12
8.11
8.10
8.09
8.08
8.07
8.06
8.05
0
1
2
3
4
5
6
7
8
Bias Voltage
Idle Cavity Resonance
versus
Diode Bias Voltage

Thesis
A385 Allender 45730
Negative resistance
parametric oscillator.